Full counting statistics of crossed Andreev reflection

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We calculate the full transport counting statistics in a three-terminal tunnel device with one superconducting source and two normal-metal or ferromagnet drains. We obtain the transport probability distribution from direct Andreev reflection, crossed Andreev reflection, and electron transfer which reveals how these processes' statistics are determined by the device conductances. The cross-correlation noise is a result of competing contributions from crossed Andreev reflection and electron transfer, as well as antibunching due to the Pauli exclusion principle. For spin-active tunnel barriers that spin polarize the electron flow, crossed Andreev reflection and electron transfer statistics exhibit different dependencies on the magnetization configuration and can be controlled by relative magnetization directions and voltage bias.

I. INTRODUCTION

In crossed Andreev (CA) reflection, a Cooper pair in a superconductor (S) is converted into an electron-hole quasiparticle pair in normal-metal terminals (N_n) or vice versa.[1,2] This process has potential applications in quantum information processing since it induces spatially separated entangled electron-hole pairs. The performance of entanglers utilizing this effect is diminished by parasitic contributions from electron transfers (ETs) between the N_n terminals. This process will be referred to as ET but is also often denoted as electron cotunneling or elastic cotunneling.

It has been suggested that the noise properties of crossed Andreev reflection can be used to distinguish it from electron transfer between the normal-metal terminals.[3] Mesoscopic transport noise also reveals information about charge carriers which is inaccessible through average current measurements. In a system with several drain terminals, i.e., current beam splitters, noise measurements show signatures of correlations between the current flow in separated terminals. The cross-correlation noise has attained interest recently since it can be utilized to study entanglement[4–16] and correlated transport.

Crossed Andreev reflection in superconductor-normal-metal systems has been experimentally studied in Refs. 17–19. In Ref. 17 the nonlocal voltage was measured in a multilayer Al/Nb structure with tunnel contacts between the superconducting Nb and the normal-metal Al layers. Current was injected through one of the normal-metal-superconductor contacts and a nonlocal voltage measured between the superconductor and the other normal metal. At injection bias voltage below the Thouless energy $E_{Th} = \hbar D / d^2$ associated with the separation d between the normal metals, positive nonlocal voltage was measured and this was interpreted as the result of dominating ET. For voltages eV above E_{Th} the nonlocal voltage changed sign, which was interpreted as a consequence of dominating CA. Subsequently, measurements reported in Ref. 18 on nonlocal voltages in Au probes connected to a wire of superconducting Al by transparent interfaces indicated that the ET contribution is larger than the contribution from CA.

The competition between CA and ET determines the sign of the nonlocal voltage and has been studied theoretically using various approaches[20–28] including the circuit theory of mesoscopic superconductivity utilized in this paper.[29,30] We will consider the linear-response nonlocal conductance G_{nl}. In superconductor-normal-metal hybrid devices where transport in one normal-metal terminal N_1 is measured in response to an applied voltage in another normal-metal terminal N_2, this quantity is defined by

$$\partial_{V_2} I_1 = -G_{nl} = -(G_{ET} - G_{CA}), \quad (1)$$

where we have introduced conductances associated with the charge-transfer processes introduced above, G_{CA} for crossed Andreev reflection, and G_{ET} for electron transfer.[20] The sign of the nonlocal conductance is determined by the competition between ET and CA. Theoretical calculations based on second-order perturbation theory in the tunneling Hamiltonian formalism predicted that the nonlocal conductance resulting from CA reflection is equivalent in magnitude to the contribution from ET.[20] Thus the induced voltage in N_1 in response to the bias on N_2 should vanish since CA and ET give currents with opposite sign, in contrast to the measurements reported in Refs. 17–19. The tunneling limit was also considered in Refs. 26, 29, and 31, and it has been found that the nonlocal conductance is in fact of fourth order in the tunneling and favors ET.

Experimental investigations of crossed Andreev reflection in superconductor-ferromagnet (S-F) structures have been reported in Refs. 32 and 33. The measurements in Ref. 32 were modeled using the theory of Ref. 20.

Experimental studies of the CA and ET noise properties can be used to determine the relative contributions of these processes to the nonlocal conductance. It was shown theoretically that CA contributes positively to the noise cross correlations, whereas ET gives a negative contribution.[3] Calculations of higher-order noise correlators or the noise dependence on spin-polarizing interfaces can reveal further information about the CA and ET processes.

We will consider the full counting statistics (FCS) which encompasses all statistical moments of the current flow.[34–36] The noise properties of ET and CA reflection thus obtained can be used to study the competition between these processes
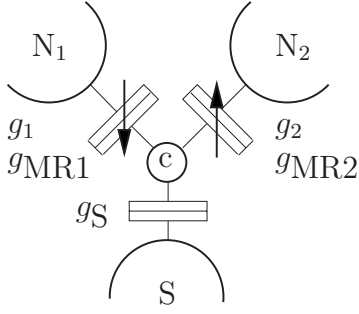
FIG. 1. Circuit theory representation of the considered beam splitter where supercurrent flows from a superconducting reservoir (S) into normal-metal drains (N_n). Tunnel barriers between cavity (c) and the drains can be spin active and are characterized by the conductances g_n and spin polarizations g_{MRn}/g_n.

and reveal information that is not accessible in the mean currents. Our calculation also determines the contribution to the noise coming from the fermion statistics (Pauli exclusion principle). Moreover, the charge-transfer probability distribution provided by FCS reveals information about the probability of elementary processes in the circuit.[37]

In this paper we calculate the FCS of multiterminal superconductor-normal-metal and superconductor-ferromagnet proximity structures, and study the currents, noise, and cross correlations associated with the various transport processes. We obtain the probability distribution for transport at one normal-metal drain and show that the probability associated with ET is larger than the probability associated with CA. For spin-active interfaces we show how spin filtering can be utilized to control the relative magnitude of the CA and ET contributions to the transport. Finally, we consider crossed Andreev reflection for spin triplet superconductors. This paper goes beyond our previous publications,[29,30,37] in that we consider different bias voltages on the normal-metal or ferromagnetic drains and discuss the effect of triplet superconductivity.

This paper is organized in the following way. In Sec. II we describe the electronic circuit and outline the formalism utilized to calculate the cumulant generating function (CGF) of the probability distributions. In Sec. III we discuss the results in the case of normal metals, and in Sec. IV we consider the spin-active connectors. Finally, our conclusions are given in Sec. V.

II. MODEL

The systems we have in mind can be represented by the circuit theory diagram (see Sec. II A) shown in Fig. 1. A superconducting source terminal (S) and normal-metal drain terminals (N_n) are connected by tunnel barriers with conductance g_n to a common scattering region which is modeled as a chaotic cavity (c). The assumptions on the cavity are that the Green's function is isotropic due to diffusion or chaotic scattering at the interfaces and that charging effects and dephasing can be disregarded. The tunnel barriers can be spin active with spin polarization g_{MRn}/g_n. We consider elastic transport at zero temperature. The superconducting termi-

nal is grounded, and biases V_n are applied to the normal terminals. We assume that $V_n \ll \Delta_0$, where Δ_0 is the gap of the superconducting terminal. In addition to the ET and CA processes described above, there can also be direct Andreev (DA) reflection between the superconductor and one normal-metal terminal, where both particles of the Andreev reflected pair are transferred into N_n. Semiclassical probability arguments show that the subgap charge current in the connector between N_1 and c in the three-terminal network in Fig. 1 has the following structure:[20,29]

$$I_1(E) = G_{CA}(V_1 + V_2) - G_{ET}(V_2 - V_1) + 2G_{DA1}V_1, \quad (2)$$

where we have introduced the conductance G_{DA1} associated with direct Andreev reflection between terminals N_1 and S. Equation (2) leads to the definition of the nonlocal conductance in Eq. (1) which shows that when $V_2 > V_1$, ET and CA give competing negative and positive contributions, respectively, to the current. The conductances in Eq. (2) will be determined in the calculation below.

A. Circuit theory

The circuit theory of mesoscopic transport is reviewed in Ref. 38 and is a suitable formalism to study proximity effects in superconducting nanostructures. The theory is developed from a discretization of the quasiclassical theory of superconductivity,[39] in combination with a theory of boundary conditions based on scattering theory.

The circuit theory is formulated in terms of the quasiclassical Green's functions of the terminals and nodes in the system. Nodes can represent small islands or lattice points of diffusive parts of the system. Under the assumptions described above, the Green's function of the spin-singlet S terminal in Fig. 1 is $\check{G}_S = \hat{\tau}_1$ where $\hat{\tau}_k$ is a Pauli matrix in Nambu space. The Green's functions $\check{G}_n$ of normal-metal terminals N_n are given by

$$\check{G}_c(E) = \begin{cases} \hat{\tau}_3\check{\tau}_3 + (\check{\tau}_1 + i\check{\tau}_2) & |E| < eV_n \\ \hat{\tau}_3\check{\tau}_3 + \mathrm{sgn}(E)\hat{\tau}_3(\check{\tau}_1 + i\check{\tau}_2) & |E| \geq eV_n, \end{cases} \quad (3)$$

where $\check{\tau}_k$ are Pauli matrices in Keldysh space.

Matrix currents $\check{I}$ describe the flow of charge, energy, and coherence between terminals and nodes through connectors; and conservation of these currents are imposed at each node. This generalized Kirchhoff law determines the Green's function of the cavity $\check{G}_c$. The balance of matrix currents $\check{I}_n$ flowing between each terminal $n = 1, 2, S$ and the cavity, including the effect of superconducting pairing in c, can be written as

$$\sum_n \check{I}_n - \left[\frac{e^2 \nu_0 \mathcal{V}_c}{\hbar} \Delta_c \hat{\tau}_1, \check{G}_c \right] = 0. \quad (4)$$

Here, Δ_c is the superconducting order parameter on the cavity, ν_0 is the density of states, and $\mathcal{V}_N$ is the volume of the cavity.[30,39] The second term on the left-hand side of the equation above induces electron-hole pairing. Since the pairing term has the same structure as the coupling to the superconducting terminal [see Eq. (5)], it gives quantitative effects

that are captured by renormalizing $e^2 \nu_0 \mathcal{V}_c \Delta_c / \hbar + g_S \rightarrow g_S$ and will be omitted in the following. Here, g_S is the superconductor tunnel conductance.

Spin-dependent transmission and reflection are described by tunneling amplitudes $t_{k,\sigma}^n$ and $r_{k,\sigma}^n$ for electrons with spin σ incident from the cavity side on the interface between the cavity and terminal n in channel k. The matrix current $\check{I}_n$ through such spin-active interfaces is[40]

$$\check{I}_n = \frac{g_n}{2}[\check{G}_n, \check{G}_c] + \frac{g_{MRn}}{4}[\{\mathbf{m}_n \cdot \overline{\boldsymbol{\sigma}} \hat{\tau}_3, \check{G}_n\}, \check{G}_c]. \quad (5)$$

Here, $g_n = g_Q \sum_{k,\sigma} |t_{k,\sigma}^n|^2$ is the tunnel conductance where $g_Q = e^2/h$ is the conductance quantum, $g_{MRn} = g_Q \sum_k (|t_{k,\uparrow}^n|^2 - |t_{k,\downarrow}^n|^2)$ is the conductance polarization, $\overline{\boldsymbol{\sigma}}$ is the vector of Pauli matrices in spin space, and the unit vector $\mathbf{m}_n$ points in the direction of the magnetization of the spin-polarizing contact. In Eq. (5) we have neglected an additional term related to spin-dependent phase shifts from reflection at the interface which can be suppressed by a thin nonmagnetic oxide layer.[41,42] The effects of spin filtering contained in the polarization g_{MRn}, which can be obtained experimentally using ferromagnetic terminals, will be studied in Sec. IV.

In systems where all the connectors are tunnel barriers described by matrix current (5), it is possible to solve Eq. (4) analytically and obtain the cavity Green's function in terms of the terminal Green's functions and the tunneling parameters. To this end, we note that it is possible to write Eq. (4) as $[\check{M}, \check{G}_c] = 0$. Employing the normalization condition $\check{G}_c^2 = 1$, the solution can be expressed in terms of the matrix $\check{M}$ as[11]

$$\check{G}_c = \check{M}/\sqrt{\check{M}^2}. \quad (6)$$

This result facilitates calculation of the cumulant generating function of the charge-transfer probability distribution in tunnel barrier multiterminal circuits.

B. Full counting statistics

Full counting statistics is a useful tool to compute currents and noise in a multiterminal structure[43] and also provides the higher statistical moments that may become experimentally accessible in these systems. Additionally, one can obtain information about the elementary charge-transport processes by studying the probability distributions.[37] The CGF $\mathcal{S}(\{\chi_n\})$ of the probability distribution is directly accessible by the Green's function method and is defined by

$$e^{-\mathcal{S}(\{\chi_n\})} = \sum_{N_n} P(\{N_n\}; t_0) \exp[-i \sum_n \chi_n N_n],$$

$$P(\{N_n\}; t_0) = \frac{1}{(2\pi)^M} \int_{-\pi}^{\pi} d^M \chi \, \exp[-\mathcal{S}(\{\chi_n\}) + i \sum_n N_n \chi_n]. \quad (7)$$

Here, $P(\{N_n\}; t_0)$ is the probability to transfer $N_1, N_2, \dots, N_n$ electrons into terminal $N_1, N_2, \dots, N_n$ in time t_0 and M is the total number of terminals in the circuit. The CGF is a func-

tion of the set of counting fields $\{\chi_n\}$ which are embedded in the Green's function at each terminal by the transformation $\check{G}_n \rightarrow e^{i\chi_n \check{\tau}_K/2} \check{G}_n e^{-i\chi_n \check{\tau}_K/2}$ where $\check{\tau}_K = \hat{\tau}_3 \check{\tau}_1$. The CGF will be determined by the following relation:[43]

$$\frac{ie}{t_0} \frac{\partial \mathcal{S}(\{\chi_n\})}{\partial \chi_n} = \int dE I_n(\{\chi_n\}), \quad (8)$$

where $I_n(\{\chi_n\})$ is the particle (counting) current through connector n in the presence of the counting fields. Our task is now to integrate this equation and obtain the CGF $\mathcal{S}(\{\chi_n\})$. Using the general solution to the matrix current conservation (6), it was found in Ref. 11 that this is possible by rewriting the counting current in terms of a derivative of $\check{M}$ with respect to the counting fields. Explicit derivation shows that

$$I_n(\{\chi_n\}) = \frac{1}{8e} \text{Tr}\{\check{\tau}_K \check{I}_n(\{\chi_n\})\} = \frac{1}{4ei} \partial_{\chi_n} \text{Tr}\{\sqrt{\check{M}^2}\}. \quad (9)$$

This result is valid also in the presence of spin-active contacts in Eq. (5). Combining Eq. (9) with Eq. (8) yields $\mathcal{S}(\{\chi_n\})$ straightforwardly.

Practical calculations of CGFs are performed by diagonalizing the matrix $\check{M}$, which allows us to express the CGF in terms of the eigenvalues of $\check{M}$,

$$\mathcal{S} = -\frac{t_0}{4e^2} \int dE \sum_k \sqrt{\lambda_k^2}. \quad (10)$$

In this equation, $\{\lambda_k\}$ is the set of eigenvalues of $\check{M}$.

We can obtain the cumulants of the transport probability distribution by successive derivatives of the CGF.[36] Specifically, we obtain the mean current from

$$I_n = -\frac{ie}{t_0} \frac{\partial \mathcal{S}(\{\chi\})}{\partial \chi_n} \bigg|_{\{\chi=0\}}. \quad (11)$$

The current noise power is given by

$$C_{m,n} = 2 \frac{e^2}{t_0} \frac{\partial^2 \mathcal{S}(\{\chi\})}{\partial \chi_m \partial \chi_n} \bigg|_{\{\chi=0\}}, \quad (12)$$

where in the multiterminal structure, the autocorrelation noise at terminal n is given by $C_{n,n}$. When $m \neq n$, Eq. (12) gives the noise cross correlations.

III. NORMAL-METAL DRAINS

In this section, we will consider the FCS of the superconducting beam splitter in Fig. 1 when the connectors are not spin polarizing and generalize previous works by taking into account a difference in drain terminal voltages $V_1 \neq V_2$. In the regime $E < eV_1, eV_2$ the only contribution to the nonlocal conductance comes from CA since we consider zero temperature. The resulting CGF was studied in Ref. 11 where it was assumed that $V_1 = V_2$. In the general case $V_1 \neq V_2$, the total CGF $\mathcal{S}$ following from Eq. (10) has one contribution from the energy range $E < eV_1, eV_2$, and if the voltages are different, there is another contribution in the energy range $eV_1 \leq E < eV_2$ (we assume $V_2 > V_1$),

$$\mathcal{S} = -\frac{t_0}{4e^2}\sum_k \left(\int_{-eV_2}^{-eV_1} + \int_{-eV_1}^{eV_1} + \int_{eV_1}^{eV_2}\right) dE \sqrt{\lambda_k^2(E)}$$
$$= \mathcal{S}_a(V_1) + \mathcal{S}_b(V_2 - V_1). \tag{13}$$

There is no contribution to transport at $|E| > eV_2$. Here, we have defined two separate contributions to the CGF that govern transport in the regime $E < eV_1, eV_2$ ($\mathcal{S}_a$) where only An-

dreev reflections (CA and DA) can occur, and the regime $eV_1 \leq E < eV_2$ ($\mathcal{S}_b$) where, in addition to Andreev reflections, ET can take place. The contribution $\mathcal{S}_a$ was calculated in Ref. 11, see Eq. (14a), where we have defined $g_\Sigma = [g_S^2 + g^2]^{1/2}$ and $g = g_1 + g_2$. The counting factors $e^{2i\chi_S - i\chi_m - i\chi_n}$ describe processes where two particles are transferred from S and one particle is counted at terminal N_m and at terminal N_n ($m, n = 1, 2$).

$$\mathcal{S}_a = -\frac{t_0 V_1}{\sqrt{2}e}\sqrt{g_\Sigma^2 + \sqrt{g_\Sigma^4 + 4g_S^2\sum_{m,n} g_m g_n (e^{2i\chi_S - i\chi_m - i\chi_n} - 1)}}, \tag{14a}$$

$$S_b = -\frac{t_0(V_2 - V_1)}{\sqrt{2}e}\sqrt{g_\Sigma^2 + 2g_1 g_2(e^{i\chi_1 - i\chi_2} - 1) + \sqrt{g_\Sigma^4 + 4g_S^2\sum_n g_n g_2(e^{2i\chi_S - i\chi_n - \chi_2} - 1) + 4g_1 g_2 g^2(e^{i\chi_1 - i\chi_2} - 1)}}. \tag{14b}$$

In Eq. (14) we show the calculated $\mathcal{S}_b$ which has contributions from electron transfer. The counting factor $e^{i\chi_1 - i\chi_2}$ describes events where an electron is transferred from N_1 to N_2. Compared to $\mathcal{S}_a$, we see that DA events between S and N_1 that would be described by counting factors $e^{2i\chi_S - 2i\chi_1}$ no longer occur. This can be understood from the electron-hole nature of Cooper pairs; see Fig. 2. Two quasiparticles, with energy $+E$ for the electron and $-E$ for the hole, constitute the Andreev reflected quasiparticle pairs in c. In the energy range considered here, $eV_1 \leq E < eV_2$, the states in N_1 are occupied, precluding DA reflection into N_1. A similar argument shows that in CA processes, the electron must be transferred into N_1 and the hole into N_2.

The nonlocal conductance $G_{nl} = G_{ET} - G_{CA}$ following from Eq. (14b) is in agreement with Ref. 29 where

$$G_{ET} = \frac{g_1 g_2}{2}\frac{2g^2 + g_S^2}{[g^2 + g_S^2]^{3/2}}, \quad G_{CA} = \frac{g_1 g_2}{2}\frac{g_S^2}{[g^2 + g_S^2]^{3/2}}. \tag{15}$$

The nonlocal conductance is dominated by ET and is of order $\mathcal{O}(g_n^4)$.[26,29] When $g/g_S \ll 1$, the nonlocal conductance
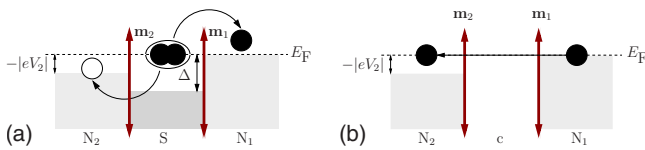

FIG. 2. (Color online) Transport processes in the three-terminal device when $eV_1 = 0$. (a) Crossed Andreev reflection: a Cooper pair from S is converted into an electron-hole pair in c by Andreev reflection, and the electron with energy $+E$ is transferred into N_1 and the hole with energy $-E$ is transferred into N_2. Tunnel barriers between the reservoirs may be spin active and are described by magnetization vectors **m** that in this paper are considered collinear. (b) Electron transfer: a particle from N_1 tunnels through the cavity c into N_2. The density of states in the cavity c is suppressed due to the proximity effect from the superconducting terminal.

vanishes due to equal probability for ET and CA as we will explicitly show by inspection of the probability distribution below. In the opposite limit that the coupling to normal terminals dominates, $g_S/g \ll 1$, the conductance for ET is to lowest order given by $g_1 g_2(1 - \delta N)/g$. Here $\delta N = (g_S/g)^2/2$ is the lowest-order correction to the low-energy density of states due to superconducting correlations. The CA conductance becomes $g_1 g_2 \delta N/g$ in this limit.

Let us now consider cross correlations. In general, CA leads to a positive contribution and ET leads to a negative contribution to the cross correlation. An additional negative contribution is induced by the Pauli exclusion principle. The cross correlation between N_1 and N_2 following from Eq. (13) is

$$C_{1,2} = 2e(V_1 + V_2)G_{CA} - 2e(V_2 - V_1)G_{ET} \tag{16a}$$

$$- \frac{10eV_1}{g_\Sigma}(G_1 - G_{nl})(G_2 - G_{nl}) \tag{16b}$$

$$+ \frac{4e(V_2 - V_1)}{g_\Sigma}[G_{CA}(G_2 + 2G_{DA2}) - G_{DA2}G_{nl}], \tag{16c}$$

where we have defined the local differential conductances $G_n = \partial_n I_n$ and the conductance for direct Andreev reflection into terminal n is $G_{DAn} = g_S^2 g_n^2/2g_\Sigma$.[29] We now focus on the competition between CA and ET. When the two normal-metal terminals are at equal voltage $V_1 = V_2$, the contribution from ET and also the term in Eq. (16) vanishes and we are left with a positive contribution to the cross correlations from CA due to the correlated particle transfer into N_1 and N_2. An additional negative contribution due to the Pauli principle in Eq. (16b) vanishes in the limit of asymmetric conductances $g_S \gg g$ or $g_{1(2)} \gg g_{2(1)}, g_S$ due to the noisy (Poissonian) statistics of the incoming supercurrent. A negative contribution from ET in Eq. (16a) is induced when there is a voltage difference between the normal-metal terminals due to the

currents with opposite signs in N_1 and N_2 resulting from this process. This demonstrates that it is possible to tune the sign of cross correlations by the voltages V_1 and V_2.[3] The contribution to $C_{1,2}$ in Eq. (16) is proportional to the voltage difference $V_1 - V_2$ and vanishes in the limit of asymmetric conductances $g_S \gg g$ or $g_{1(2)} \gg g_{2(1)}, g_S$.

It is interesting to compare $\mathcal{S}_b$ with the corresponding CGF when S is in the normal state,

$$
\mathcal{S}_b = -\frac{t_0(V_2 - V_1)}{2e}
$$
$$
\times \sqrt{(g_1 + g_S + g_2)^2 + 4g_2 g_1 e^{i\chi_1 - i\chi_2} + 4g_2 g_S e^{i\chi_S - i\chi_2}}.
$$
(17)

Here we see a contribution due to transport between N_1 and N_2 that is similar to the one outside the double square root in Eq. (14b). Superconductivity leads to the double square root in Eq. (14b) that takes into account the correlation of transport through c by Andreev reflections and ET. The complicated dependence on the counting fields in Eq. (14b) precludes a simple interpretation of $\mathcal{S}_b$ in terms of the probabilities of elementary charge-transfer processes. However, when $g_S \gg g$ or $g_{1(2)} \gg g_{2(1)}, g_S$ we can expand the square roots in $\mathcal{S}_b$ and obtain the CGF

$$
\mathcal{S}_b = -\frac{t_0(V_2 - V_1)}{2g_\Sigma^3 e}[g_1 g_2(g_S^2 + 2g^2)e^{i\chi_1 - i\chi_2} + g_S^2 g_1 g_2 e^{2i\chi_S - i\chi_1 - i\chi_2}
$$
$$
+ g_S^2 g_2^2 e^{2i\chi_S - 2i\chi_2}].
$$
(18)

In this limit the CGF describes independent CA, ET, and DA Poisson processes. The prefactors determine the average number of charges transferred by each process in time t_0.

To illustrate the physics described by $\mathcal{S}_b$ in the limit introduced above, let us examine the probability distribution obtained by the definition in Eq. (7). If we consider the current response in N_1 to a voltage in N_2, we can consider that $V_1 = 0$ and the only contribution to the total CGF $\mathcal{S}$ comes from $\mathcal{S}_b$. The normalized probability distribution for the transport at terminal N_1 following from Eq. (18) then becomes

$$
P(N_1; t_0) = e^{-\bar{N}_1 g_\Sigma^2/g^2} \sum_{\substack{k \geq |N_1| \\ k + N_1 \text{ even}}} \left(\bar{N}_1 \frac{g_S^2}{2g^2}\right)^{k + N_1/2}
$$
$$
\times \left[\bar{N}_1\left(\frac{g_S^2}{2g^2} + 1\right)\right]^{k - N_1/2}
$$
$$
\times \left[\left(\frac{k + N_1}{2}\right)! \left(\frac{k - N_1}{2}\right)!\right]^{-1}.
$$
(19)

Here we have defined the mean number of particles transferred in time t_0,

$$
\bar{N}_1 = \frac{|I_1| t_0}{e} = \frac{V_2 t_0}{e} \frac{g_1 g_2}{g(1 + g_S^2/g^2)^{3/2}}.
$$
(20)

Equation (19) describes a joint probability distribution for CA and ET processes with Poissonian statistics and is constrained such that the number of CA events described by the
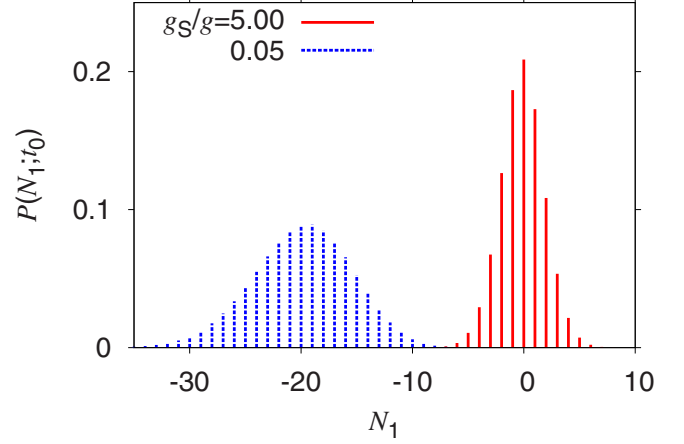

FIG. 3. (Color online) Probability distribution for transport of N_1 electrons into terminal N_1, $P(N_1; t_0)$ [Eq. (19)]. We show distributions for two different values of the parameter $g_S/g = 5.00$ (red [gray] solid impulses) and 0.05 (blue [dark gray] dotted impulses). We have chosen the parameter $\alpha = g_1 g_2 V_2 t_0/eg = 20$, which gives the mean value $\bar{N}_1$ for small g_S/g; see Eq. (20).

weight $g_S^2/2g^2$ subtracted by the number of ET events described by the weight $g_S^2/2g^2 + 1$ is N_1 as required. When $g_S/g \gg 1$, the mean number of particles transferred vanishes according to Eq. (20) and probability distribution (19) is symmetric around $N_1 = 0$. This means that the average current vanishes since the probabilities for ET and CA are equal. In general, the probability distribution has its maximum for negative N_1, i.e., ET is more probable than CA reflection. In Fig. 3 we have plotted probability distribution (19) for different values of g_S/g. For small ratios g_S/g, ET dominates and the probability distribution is centered at a negative value for N_1. As expected, we see that the center of the probability distribution (mean number of particles transferred) is shifted from a negative value toward zero with increasing g_S/g. The width of the distribution, described by the autocorrelation noise $C_{1,1}$ [see Eq. (12)], decreases with increasing g_S/g.

IV. SPIN-ACTIVE CONNECTORS

Qualitatively, the effect of spin-polarizing interfaces on the competition between CA and ET processes in S-F systems can be understood as follows.[20] The ET process is favored when magnetizations of ferromagnetic leads are parallel since the same spin must traverse both the interfaces between c and the ferromagnets. On the other hand, CA reflection is favored in an antiparallel configuration since two particles with opposite spins must traverse the interfaces. This behavior was experimentally observed in Refs. 32 and 33 where ferromagnetic Fe probes were contacted to a superconducting Al wire.

The FCS of a beam splitter with spin-active contacts and $V_1 = V_2$ was considered in our previous paper[37] and constitutes $\mathcal{S}_a$; see Eq. (21a). In this case the only transport processes are DA and CA reflection, and we found that CA

is enhanced in an antiparallel alignment of the magnetizations as expected.

When $V_2 > V_1$, there can also be ET, and an additional effect is spin accumulation at the node. With collinear magnetizations [sign of g_{MRn} describes magnetization directions up (positive) or down (negative) along the z quantization axis], we find that in the regime $eV_1 < E < eV_2$, $\mathcal{S}_b = \mathcal{S}_{b+} + \mathcal{S}_{b-}$ where $\mathcal{S}_{b\sigma}$ is given below in Eq. (21b),

$$\mathcal{S}_a = -\frac{t_0 V}{\sqrt{2}e}\sqrt{1 + \sqrt{1 - \frac{4g_{MR}^2}{g_\Sigma^4}(g_S^2 + g^2) + \frac{4g_S^2}{g_\Sigma^4}\sum_{m,n}(g_m g_n - g_{MRm}g_{MRn})(e^{2i\chi_S - i\chi_m - i\chi_n} - 1)}}, \tag{21a}$$

$$\mathcal{S}_{b\sigma} = -\frac{t_0 g_\Sigma (V_2 - V_1)}{2\sqrt{2}e}\left\{1 + \frac{2}{g_\Sigma^2}(g_1 + \sigma g_{MR1})(g_2 + \sigma g_{MR2})(e^{i\chi_1 - i\chi_2} - 1) + \left[1 - \frac{4g_{MR}^2}{g_\Sigma^4}(g_S^2 + g^2) + \frac{4g_S^2}{g_\Sigma^4}\sum_n (g_n - \sigma g_{MRn})(g_2 + \sigma g_{MR2})\right.\right.$$
$$\times (e^{2i\chi_S - i\chi_n - i\chi_2} - 1) + \left.\left.\frac{4(g - g_{MR})^2}{g_\Sigma^4}(g_1 + \sigma g_{MR1})(g_2 + \sigma g_{MR2})(e^{i\chi_1 - i\chi_2} - 1)^{1/2}\right]\right\}^{1/2}. \tag{21b}$$

Here we have redefined $g_\Sigma = (g_S^2 + g^2 + g_{MR}^2)^{1/2}$ and introduced $g_{MR} = g_{MR1} + g_{MR2}$. The expression for S_b reduces to the result for nonpolarizing contacts, Eq. (14), in the limit that $g_{MRn} \rightarrow 0$. The two terms $\mathcal{S}_{b\sigma}$ correspond to the two possible directions of the spin(s) involved in the charge-transfer processes. The spin-dependent conductance for a spin up (down) is $g_n + (-)g_{MRn}$. In ET, one spin must traverse the two spin-active interfaces; thus the counting factor for spin σ is proportional to the weight $(g_1 + \sigma g_{MR1})(g_2 + \sigma g_{MR2})$. The two spin channels are independent. The two opposite spins of an Andreev reflected quasiparticle pair can be CA reflected into terminals with different polarizations g_{MR1} and g_{MR2} according to the prefactor $(g_1 - \sigma g_{MR1})(g_2 + \sigma g_{MR2})$, and each possibility for the directions of the two spins gives an independent contribution to the CGF $\mathcal{S}_b$.

In the limit that $g_S \gg g$ or $g_{1(2)} \gg g_{2(1)}, g_S$ ($g_{MRn} \le g_n$ by definition), we can expand the double square roots and perform the summation over $\mathcal{S}_{b\sigma}$ which yields

$$\mathcal{S}_b = -\frac{t_0(V_2 - V_1)}{2eg_\Sigma^3}\{[g_\Sigma^2 + (g - g_{MR})^2](g_1 g_2$$
$$+ g_{MR1}g_{MR2})e^{i\chi_1 - i\chi_2} + g_S^2(g_1 g_2 - g_{MR1}g_{MR2})e^{2i\chi_S - i\chi_1 - i\chi_2}$$
$$+ g_S^2(g_2^2 - g_{MR2}^2)e^{2i\chi_S - 2i\chi_2}\}. \tag{22}$$

The nonlocal conductance following from Eq. (22) is given by

$$G_{ET} = (g_1 g_2 + g_{MR1}g_{MR2})\frac{[g_\Sigma^2 + (g - g_{MR})^2]}{2g_\Sigma^3}, \tag{23a}$$

$$G_{CA} = (g_1 g_2 - g_{MR1}g_{MR2})\frac{g_S^2}{2g_\Sigma^3}. \tag{23b}$$

This immediately demonstrates that ET is favored in a parallel configuration of the magnetizations ($g_{MR1}g_{MR2} > 0$) as the same spin in this case tunnels through both interfaces. On the other hand, CA reflection is favored by antiparallel mag-

netizations ($g_{MR1}g_{MR2} < 0$) since the opposite spins of a singlet tunnel through different interfaces. These qualitative features are in agreement with Ref. 20.

The cross correlation following from Eq. (22) is

$$C_{1,2} = 2e(V_2 + V_1)G_{CA} - 2e(V_2 - V_1)G_{ET}. \tag{24}$$

The sign of $C_{1,2}$ can now be tuned by two experimental control parameters: the bias voltages through the prefactors in Eq. (24) and the relative magnetization direction that determines the magnitudes of G_{ET} and G_{CA}.

In the energy range $V_1 < E < V_2$ we are in this setup measuring the energy of the quasiparticles involved in crossed Andreev reflection; see Fig. 2. Since the electronlike quasiparticle with energy $+E$ must flow into N$_1$, and the $-E$ hole-like quasiparticle must flow into N$_2$, this means that the entanglement in the energy degree of freedom of Andreev reflected quasiparticle pairs has collapsed.

Triplet superconductivity

Superconducting correlations with triplet pairing symmetry in the spin space can be induced by magnetic exchange fields in singlet superconductor heterostructures. This effect has attained considerable interest and has recently been experimentally demonstrated (see Refs. 44–46 and references therein). The different spin-space symmetry of the induced electron-hole correlations opens interesting experimental applications where, e.g., superconducting correlations can propagate through a strong ferromagnetic material.[47,48] We have studied the FCS when S is a source of spin triplet quasiparticle pairs, and in this section we summarize our results for collinear magnetizations when $V_1 = V_2$.

The CGF for the spin triplet Cooper pairs with $S_z|\Psi\rangle = 0$, where S_z is the spin operator along the z axis and $|\Psi\rangle$ is the spin part of the Cooper pair wave function, is identical to the CGF for conventional spin-singlet superconductors. We showed in Ref. 37 that CGF (21a) reveals the entangled nature of the quasiparticle pairs. The $S_z|\Psi\rangle = 0$ spin triplet states

are also one of the maximally entangled Bell states which implies that the magnetization dependence for CA is the same for singlet and triplet in the collinear case. This result can be shown also straightforwardly by computing the two-electron tunneling probability $p_{1,2}$ which is proportional to

$\langle\Psi|(g_1+g_{MR1}\bar{\sigma}_3)\otimes(g_2+g_{MR2}\bar{\sigma}_3)|\Psi\rangle+1\leftrightarrow2$.

The triplet states where $S_z|\Psi\rangle=\pm\hbar\ |\Psi\rangle$ give rise to a different dependence on the magnetization configurations in the CGF since the quasiparticle pairs are not in spin entangled states but rather in product states. We obtain the CGF

$$\mathcal{S}_{\parallel}=-\sum_{\sigma}\frac{t_0 V}{2\sqrt{2}e}\sqrt{1+\sqrt{1-\frac{4g_{MR}^2}{g_{\Sigma}^4}(g_S^2+g^2)+\frac{4g_S^2}{g_{\Sigma}^4}(g_1+\sigma g_{MR1})(g_2+\sigma g_{MR2})(e^{2i\chi_S-i\chi_m-i\chi_n}-1)}}. \tag{25}$$

Compared to singlet case (21), we see that the CA counting prefactor factorizes as a result of the nonentangled product state of the quasiparticle pairs. This magnetization dependence can be recovered also by calculating the two-particle tunneling probability $p_{1,2}$ as discussed above.

V. CONCLUSION

We have calculated the full counting statistics of multiterminal tunnel-junction superconductor-normal-metal and superconductor-ferromagnet beam splitter devices and studied the resulting currents and cross correlation. The probability distribution for transport at a normal-metal drain contact demonstrates that the probability for electron transport between two normal-metal terminals is larger than the probability for crossed Andreev reflection. A finite voltage difference between the normal-metal contacts introduces competing contributions to the cross correlations from electron transport between normal terminals and crossed Andreev reflection. Finally, we have shown how spin-active contacts act as filters for spin and calculated the cumulant generating function. The sign of the cross correlation due to the competing contributions from electron transport between drain terminals and crossed Andreev reflection can in this case be determined by two external control parameters, i.e., bias voltages and the relative magnetization orientation. Finally, we make some remarks about the counting statistics in the case of spin triplet superconductors.

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